ISI – Bangalore Center – B Math - Physics II – End Semestral Exam
Date: 4 November 2015. Duration of Exam: 3 hours
Total marks: 50

ANWER ALL QUESTIONS

Q 1 [Total Marks:3+3+2+2=10]

A THICK walled insulated metal chamber contains n_i moles of helium at high pressure P_i. It is connected through an insulated valve with a large almost empty thermally insulated gasholder kept at constant pressure P very close to the atmospheric pressure. The valve is opened slightly and helium is allowed to flow SLOWLY and adiabatically into the gasholder until the flow stops due to equalization of pressure. Let n_f be the final number of moles in the CHAMBER.

a.) Show that $\frac{n_f}{n_i} = \frac{h-u_i}{h-u_f}$ where $h = u + Pv$, where u and v are the molar internal energy and molar volume of helium in the GAS HOLDER respectively, and u_i, u_f are the initial and final molar energies of helium in the CHAMBER respectively.

b.) Assuming helium obeys ideal gas laws show that $\frac{n_f}{n_i} = \left(\frac{P}{P_i}\right)^{\frac{1}{\gamma}}, \gamma = \frac{c_p}{c_v}$

c.) Will the answer in part a) change if helium is allowed to flow suddenly instead of slowly? Justify your answer.

d.) Will the answer in b.) change if helium is allowed to flow suddenly instead of slowly? Justify your answer.

Q 2 [Total Marks: 2+2+4=8]

A body is initially kept at thermal equilibrium with a heat reservoir at temperature T_1. It is then placed in a refrigerator which, working in cycles, reduces the temperature to T_2 by dumping heat into the reservoir. Assume that the entropy of the body being cooled changes from S_1 to S_2.

a.) What is the change of entropy of the refrigerator?
b.) What is the change of entropy of the reservoir?
c.) Show that the minimum amount of energy that has to be supplied to the refrigerator is $T_1(S_1 - S_2) - C(T_1 - T_2)$ where C is the specific heat of the body

Q3. [4+4+2=10]

a.) Describe the process of Joule Thompson expansion for a gas (not necessarily an ideal gas) and show that enthalpy is conserved in this process.

b.) Explain the concept of inversion curve using T-P diagram and with the help of typical curves of constant enthalpy. Show that the Joule Thompson process may not always lower the temperature of the gas undergoing expansion.

c.) The inversion temperature of nitrogen and helium are 607 K and 43 K respectively. If the same number of moles of nitrogen and helium are separately put through a J-T process with lowering of pressure from 200 atm to 1 atm, at typical room temperature in summer in Bangalore, which one will cool and which one will get hot?

Q 4. [2+4+3+3=12]

Consider a model of rubber band as a chain of N links of negligible width, with each of the links being of length a and each link capable of pointing either to the left or to the right. Note that if the rubber band is thermodynamically described by its length L under tension f at temperature T, then there may be many microstates which can correspond to this macro state.

a.) Show that the number of states is given by $\Omega(N,L,a)=\dfrac{N!}{\left(\dfrac{N}{2}-\dfrac{L}{2a}\right)!\left(\dfrac{N}{2}+\dfrac{L}{2a}\right)!}$.

[Hint: if there are N_R (N_L) links pointing to the right (left), then express N_R (N_L) in terms of N, L, and a .]

b.) Show that in the limit of large N and $(L/Na)<<1$, the entropy of the band can be written as

$$(S/k)=N\left[\ln 2-\frac{1}{2}\left(\frac{L}{Na}\right)^2\right].$$

c.)Show that for a rubber band with length L under tension f, $f=-T\left(\dfrac{\partial S}{\partial L}\right)_U$ where U is energy of the band .

d.) Hence show that in the same limit of large N and $(N/La)<<1$, the equation of state is given by $f=\dfrac{LkT}{Na^2}$. What can you conclude from this above the thermal expansion of a rubber band under constant tension (give quantitative reasoning to justify your answer)

Q5.[2+3+5 =10]

a.) Suppose we conduct Newton's rings experiment with monochromatic light of wavelength λ, while slowly raising the lens vertically above the plate. By how much do we need to raise the lens for the center to have a dark spot again?

b.) After the lens has been raised as in part a.) , what will be the radii of the next two dark rings? Express the answers in terms of λ and R where R is the radius of the lens.

c.) Consider the formation of Newton's rings when two closely spaced wavelengths λ, and $\lambda + \Delta\lambda$ are present. Show that the value of m, the order of interference, for which the bright and dark rings will coincide, is given by $\frac{1}{2}\frac{\lambda}{\Delta\lambda}$